

MATH 121 - Test Assignment #1 (Solutions)

- ① Given the supply function $p = 0.75q + 50$
and the demand function $p = -0.5q + 100$

(i) Equilibrium occurs when supply = demand

$$\Rightarrow 0.75q + 50 = -0.5q + 100$$

$$\Rightarrow 0.75q + 0.5q = 100 - 50$$

$$\Rightarrow 1.25q = 50$$

$$\Rightarrow q = \frac{50}{1.25} = 40 \text{ units}$$

(ii) Market-clearing price is
using the supply function

$$p = (0.75)(40) + 50 = \$80$$

OR using the demand function

$$p = (-0.5)(40) + 100 = \$80$$

② Let $P(q) = aq^2 + bq + c$

(i) when $q = 0$, then $P = -\$120$

$$\text{so } -120 = a(0^2) + b(0) + c \Rightarrow c = -120 \quad \text{--- (1)}$$

(ii) when $P = 0$, then $q = 20$ or $q = 60$

$$\text{so } 0 = a(20^2) + b(20) + c \Rightarrow 400a + 20b + c = 0 \quad \text{--- (2)}$$

$$\text{and } 0 = a(60^2) + b(60) + c \Rightarrow 3600a + 60b + c = 0 \quad \text{--- (3)}$$

Substitute (1) into (2)

$$400a + 20b - 120 = 0$$

$$(\div 20) \Rightarrow 20a + b - 6 = 0$$

$$\Rightarrow 20a + b = 6 \quad \text{--- (4)}$$

Substitute (1) into (3)

$$3600a + 60b - 120 = 0$$

$$(\div 60) \Rightarrow 60a + b - 2 = 0$$

$$\Rightarrow 60a + b = 2 \quad \text{--- (5)}$$

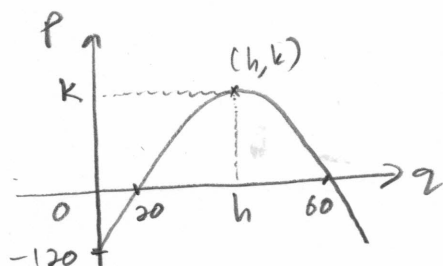
$$(5) - (4): \quad 40a = -4 \Rightarrow a = -\frac{4}{40} = -\frac{1}{10}$$

Substitute into (4): $(20)(-\frac{1}{10}) + b = 6$
 $\Rightarrow -2 + b = 6 \Rightarrow b = 6 + 2 = 8$

OR Substitute into (5): $(60)(-\frac{1}{10}) + b = 2$
 $\Rightarrow -6 + b = 2 \Rightarrow b = 2 + 6 = 8$

$$\therefore P(q) = -\frac{1}{10}q^2 + 8q - 120 \quad ; \quad q \geq 0$$

(ii)



The maximum profit occurs at the vertex (h, k)

The optimum number of cups the cafe owner should brew to maximise his profit is

$$h = -\frac{b}{2a}$$

From $P = -\frac{1}{10}q^2 + 8q - 120$, we have $a = -\frac{1}{10}$, $b = 8$, $c = -120$

so $h = -\frac{8}{2(-\frac{1}{10})} = \frac{8}{\frac{1}{5}} = 40$ cups

The maximum profit is

$$\begin{aligned} k &= c - ah^2 = -120 - (-\frac{1}{10})(40^2) \\ &= -120 + \frac{1600}{10} = \$40 \end{aligned}$$

$$(3) \quad p = \frac{aq + b}{q + c}$$

(i) when $p = \$20$, $q = 0$

so $20 = \frac{a(0) + b}{0 + c} \Rightarrow 20 = \frac{b}{c} \Rightarrow b = 20c \quad - (1)$

(ii) when $p = \$13$, $q = 20$

so $13 = \frac{a(20) + b}{20 + c} \Rightarrow 20a + b = 13(20 + c) = 260 + 13c \quad - (2)$

(iii) when $q \rightarrow \infty$, $p \rightarrow \$6$

so $\lim_{q \rightarrow \infty} \frac{aq + b}{q + c} = \lim_{q \rightarrow \infty} \frac{q \left(\frac{aq + b}{q} \right)}{q \left(\frac{q + c}{q} \right)} = \lim_{q \rightarrow \infty} \frac{q \left(a + \frac{b}{q} \right)}{q \left(1 + \frac{c}{q} \right)}$
 $= \lim_{q \rightarrow \infty} \frac{a + \frac{b}{q}}{1 + \frac{c}{q}} = \frac{a + 0}{1 + 0} = a \Rightarrow a = 6 \quad - (3)$

Substitute (1) and (3) into (2):

$$(120)(6) + 20c = 260 + 13c$$

$$\Rightarrow 120 + 20c = 260 + 13c$$

$$\Rightarrow 20c - 13c = 260 - 120$$

$$\Rightarrow 7c = 140 \Rightarrow c = \frac{140}{7} = 20$$

Substitute into (1):

$$b = (20)(20) = 400$$

$$p = \frac{6q + 400}{q + 20} ; q \geq 0$$

④ Given $p(q) = 65000 - 100q$; $1 \leq q \leq 40$

and $q(w) = 0.2w + 1.6\sqrt{w}$; $w \geq 0$

(i) so $p(q(w)) = 65000 - 100(0.2w + 1.6\sqrt{w})$
 $= 65000 - 20w - 160\sqrt{w}$; $w \geq 0$

(ii) From $p = 65000 - 100q$

$$\Rightarrow 100q = 65000 - p$$

$$\Rightarrow q(p) = \frac{65000 - p}{100}$$
$$= 650 - \frac{1}{100}p$$

Since p is linear,

when $q=1$, then $p = 65000 - 100 = 64900$

when $q=40$, then $p = 65000 - 100(40) = 61000$

so the domain for the inverse function

is $\$61,000 \leq p \leq \$64,900$